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RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

V - EFFECTIVENESS OF AIR COOLING OF HOLLOW BLADES

By Lincoln Wolfenstein, Robert L. Maxwell
and John S. McCarthy

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SUMMARY

An analysis and discussion of the cooling of hollow turbine blades by forcing air through a passage in the blade and out the tip is presented. An equation for the temperature distribution along the blade is derived. By use of this equation and experimental stress-rupture data, a relation is established between allowable gas temperature, blade speed, and a parameter combining blade dimensions, cooling-air flow, and heat-transfer coefficient from the hot gas to the blade. Three limits to the possible amount of cooling are discussed and equations are derived to show the effects of the various blade and turbine dimensions and flow conditions on the two important limits, dilution and cooling-air Mach number. Three representative turbines were selected and compared with respect to cooling effectiveness as a function of dilution and Mach number. It was found that an increase in allowable gas temperature from 150° to 650° F, depending on turbine dimensions and design speed, might result from air cooling. Turbines with long thin blades and large hot-gas flows, which cannot be effectively cooled by indirect methods such as rim cooling, can be effectively cooled by air cooling. Increases in gas temperature of approximately 500° F are made possible by air cooling of hollow turbine blades compared to about 200° F for indirect cooling methods.

INTRODUCTION

The power and efficiency of current designs of compressor-turbine units are limited by the allowable peak gas temperature, which is in turn restricted by the strength of the first-stage rotor blades in the turbine. An investigation of methods of cooling turbine blades is being conducted at the NACA Cleveland laboratory in order to increase the range of possible gas temperatures.

Turbine blades may be indirectly cooled by conducting heat from the blade to the blade tip or root, or directly cooled by forcing a cooling fluid through a passage within the blade.

In the first and second reports in a series of reports on cooling of gas turbines (references 1 and 2), the possibilities of indirect cooling were investigated on a theoretical basis, and it was found that the benefits of indirect cooling are limited to increases in allowable gas temperature of the order of 200° F for most turbines. Furthermore, it was indicated that these benefits are much less for high-power turbines because of the long blades and the high specific mass flows that characterize them. A study of cooling of turbines (reference 3) showed that considerably greater increases in allowable temperature are made possible with direct cooling, with either water or air as the coolant, than by indirect cooling for a particular turbine. Consequently a third report of the series was prepared, which analyzed liquid cooling in detail and indicated the large benefits available from direct liquid cooling using coolant passages within the blade.

Recognizing the mechanical difficulties involved in liquid cooling, the present report considers direct cooling achieved by forcing air through hollow passages in the blades and out the blade tips. The analysis and the method of presentation are similar to those of reference 2, where the benefits of blade cooling were measured by the increase in allowable effective gas temperature for a constant blade Mach number and life. Quantitative calculations are made for three typical turbines and the two factors that may possibly limit the amount of cooling are investigated: dilution of the hot gas with cooling air, and the Mach number of the cooling air within the blade. For each turbine, the amount of cooling air was varied from zero to the amount corresponding to either hot-gas dilution of 50 percent or a cooling-air Mach number within the blade of 1.0, whichever was lower. For each turbine the cooling was varied by this amount for blade-tip Mach numbers of 0.4 to 0.8. In addition the effects of variations in turbine dimensions and gas-flow conditions on the effectiveness of cooling are considered.

ANALYSIS

The method of analysis is quite similar to that in reference 2. A method of finding the limiting blade tip speed V_{max} and the corresponding limiting tip Mach number index M_{max} at which any blade may be run without failure due to centrifugal stress, is presented for given: (a) blade material, (b) blade life, (c) effective gas temperature t_e, (d) mass velocity of cooling air G_a, (e) values of two turbine parameters Y and Q, and (f) ratio of blade length to tip radius L/r_t. (All symbols used are summarized in appendix A.) The method consists in combining experimental stress-rupture data for a blade material with expressions for the stress and temperature distributions in the blade.

In addition, equations are derived for the Mach number of the cooling air inside the blade and for the dilution, which is defined as the ratio of cooling-air mass flow to the mass flow of hot gases.

Assumptions. - In the derivation of expressions for the temperature and stress distributions and for the Mach number and the dilution formulas, the following simplifying assumptions are made:

1. The blade and the blade passage are of uniform cross-sectional area and perimeter. A diagram of a typical blade is given in figure 1.

2. The wall thickness is negligible insofar as heat flow is concerned. Thus, no temperature drop occurs through the wall and, at any point in the blade wall, heat may flow only in a direction perpendicular to the surface. A preliminary calculation indicated that heat conduction in the radial direction significantly affects, at most, the temperatures over that 10 percent of the blade nearest the root. The magnitude of the effect of radial heat conduction depends upon the amount of heat conducted away at the blade root for the particular installation.

3. Bending stresses in the blade are negligible and therefore the stress is constant over any blade cross section and equal to the calculated centrifugal stress at that cross section.

4. The cooling-air temperature at the blade root $t_{a,r}$ is 300° F.

5. Radiation from other hot surfaces in the turbine is ignored; the quantitative effect of radiation on the effectiveness of cooling was approximated and found to be negligible.

6. The outside heat-transfer coefficient h_o and the effective gas temperature t_e are constant over the blade outer surface; the inside heat-transfer coefficient h_i is constant over the inner surface of the blade.

7. The following equations for h_o and h_i, although based on limited data (reference 6 and data supplied by the General Electric Company), are assumed to hold true throughout the report:

$$h_o = CG_g^{0.685} p_o^{-0.315}$$

$$h_i = 0.020\left(\frac{D_i G_a}{\mu_a}\right)^{0.8} \frac{k_a}{D_i}$$

where

$$D_i = \frac{4A_i}{p_i}$$

and

C	constant, (C = 0.0323 Btu/(°F)(hr$^{0.315}$)(ft$^{0.315}$)(lb$^{0.685}$))
G_g	mass velocity of hot gas relative to blades, lb/(hr)(sq ft)
p_o	cross-sectional perimeter of outside of blade, (ft)
D_i	hydraulic diameter of cooling passage, (ft)
G_a	mass velocity of cooling air relative to blades, lb/(hr)(sq ft)
μ_a	absolute viscosity of cooling air, lb/(hr)(ft)
k_a	thermal conductivity of cooling air, Btu/(hr)(sq ft)(°F/ft)
A_i	cross-sectional area of cooling passage, (sq ft)
p_i	cross-sectional perimeter of cooling passage, (ft)

Temperature distribution. - The blade-metal temperature t_m at any distance x from the blade root is found from a heat balance. (See appendix B.) For a given value of the effective hot-gas temperature t_e, the blade-metal temperature t_m as a function of x/L is found to depend only on a turbine parameter Q and a cooling variable S:

$$t_e - t_m = \frac{1}{S + 1} (t_e - t_{a,r})\, e^{-\left[S^{1.25}\, Q\left(\frac{1}{S + 1}\right) \frac{x}{L}\right]} \tag{1}$$

where

$$S = \frac{h_o p_o}{h_i p_i}$$

The cooling variable S can also be expressed in terms of a second turbine parameter Y and the mass velocity G_a of the cooling air

$$S = Y\, G_a^{-0.8} \tag{2}$$

Thus if t_e and G_a are given, the temperature distribution along the blade depends only on Y and Q, which are given as functions of the turbine-blade dimensions and outside heat-transfer coefficient by equations (11) and (12) of appendix B.

Stress distribution. - The centrifugal stress s in the blade at any point x is found by integrating the centrifugal load from the point x to the blade tip

$$s = \frac{\rho_m V^2\left[1 - \left(\frac{r}{r_t}\right)^2\right]}{288g} = \frac{\rho_m V^2}{288g} \left\{1 - \left[1 - \frac{L}{r_t}\left(1 - \frac{x}{L}\right)\right]^2\right\} \tag{3}$$

Consequently, for given values of the ratio of blade length to tip radius L/r_t and the metal density ρ_m, the stress as a function of x/L depends only on the tip speed V.

Limiting tip speed. - The expression for centrifugal-stress distribution (equation (3)) may be combined with experimental stress-rupture data for any given blade material to provide a family of allowable temperature curves for which the tip speed V is the parameter. The limiting tip speed V_{max} for any given conditions can then be found by superimposing a plot of the blade-temperature distribution for the given conditions on the allowable-temperature curves. The point of tangency of the temperature-distribution curve and an allowable-temperature curve indicates the maximum allowable tip speed and also the critical point of the blade where the actual temperature equals the allowable temperature, and fracture would be expected. If the temperature-distribution curve is so flat that no point of tangency occurs, the critical point is at the blade root. This procedure is illustrated in figure 2.

Limiting tip Mach number index. - The limiting tip speed V_{max} may be converted to the limiting tip Mach number index M_{max} as follows:

$$M_{max} = \frac{V_{max}}{a_g} = \frac{V_{max}}{\sqrt{\gamma g R(t_e + 460)}}$$

where

a_g sonic velocity of hot gases at temperature t_e, (ft/sec)

γ ratio of specific heats

R gas constant, 53.5 (ft-lb)/(lb)($^{\circ}$R)

Dilution and cooling-air Mach number. - With the method just explained, the effectiveness of air cooling of hollow blades depends only on the values of S, Q, and L/r_t. Because variations in Q and L/r_t within the probable range of values have only a slight effect on the results, the effectiveness of the cooling is almost completely determined by the single factor S, which depends on the cooling-air mass flow. Two factors that may limit the amount of cooling possible are the dilution and the Mach number of the cooling air; it is therefore desirable to obtain expressions for these two limits in terms of S.

Dilution or the ratio of cooling-air mass flow m_a to hot-gas mass flow m_g is given by

$$\frac{m_a}{m_g} = \frac{G_a A_i}{m_g}$$

and may be expressed in terms of S. (See appendix C.)

$$\frac{m_a}{m_g} = 1512S^{-1.25}\ C^{1.25}\ G_g^{-0.144\left(\frac{A_o}{A_b}\right)}\ \alpha^{1.25}\ A_o^{0.25}\ \beta^{-1.5}\ p_o^{-0.644} \tag{4}$$

where

A_b minimum hot-gas flow area per blade, (sq ft)

A_o cross-sectional area of outside of blade, (sq ft)

α cross-sectional area ratio, A_i/A_o

β cross-sectional perimeter ratio, p_i/p_o

The second possible limit to the practicable amount of air cooling is the compressibility of the air at or near the speed of sound. For a given blade, the amount of air that may flow through the cooling passage reaches a maximum at a Mach number of 1.0. The Mach number of the cooling air M_a is defined as the ratio of the cooling-air velocity V_a to the critical velocity a_a of the cooling air:

$$M_a = \frac{V_a}{a_a} = \frac{V_a}{\sqrt{\frac{2\gamma}{\gamma + 1} gR\ T_a}} = \frac{G_a/\rho_a}{3600\sqrt{\frac{2\gamma}{\gamma + 1} gR\ T_a}}$$

and may be expressed as a function of S. (See appendix C.)

$$\epsilon M_a = 0.421 S^{-1.25}\ C^{1.25}\ G_g^{\ 0.856}\ \alpha^{0.25}\ A_o^{\ 0.25}\ \beta^{-1.5}\ p_o^{\ -0.644}\ P^{-1}$$

$$T_a^{\ 0.5}\left[\frac{2\gamma g}{(\gamma + 1)R}\right]^{-0.5} \tag{5}$$

where

T_a total temperature of cooling air, $^{\circ}R$

ρ_a density of cooling air, (lb/cu ft)

ϵ ratio of cooling-air static density to total density

$$\epsilon = \left(1 - \frac{\gamma - 1}{\gamma + 1} M_a^{\ 2}\right)^{\frac{1}{\gamma - 1}}$$

P total pressure of cooling air, (lb/sq ft) absolute

APPLICATION OF ANALYSIS

The foregoing analysis is general and applies to any hollow blade having a uniform cross section and comparatively thin walls. Quantitative results are obtained by applying this method of analysis to three representative turbines, the dimensions of which are given in table I. Full-scale sketches of these blades are given in figure 3. In addition, the gas-flow conditions, taken from test results for each of the turbines at rated conditions and the external heat-transfer coefficient calculated from equation (13) (appendix C) were used in the calculations. These values are given in table II.

The values of A_i and p_i, and consequently the values of α and β, were determined by assuming that each blade had a wall thickness of 0.015 inch. For this thickness, the temperature drop across the blade wall is, at most, 2 percent of the temperature drop across the gas film and consequently may be neglected. In the case of turbine B, however, a large temperature gradient may exist through the extended blade trailing edge as indicated in reference 5. Failure of the blade at the trailing edge must be prevented in some way if the calculations made are to be considered applicable to the blade of turbine B shown in figure 3. Although the three turbines used to demonstrate the method of calculation are quite different and represent separate turbine types, none could be considered extreme in any factor of design.

The ratio of specific heats γ was taken as 1.31 for the hot gas and 1.40 for the cooling air.

Cooling-air Mach number. - In the calculation of the cooling-air Mach number, the total pressure of the cooling air in the blade was taken as an average of the total pressure of the hot gas ahead of the blades and the total pressure behind the blades; the total temperature of the cooling air was assumed equal to the effective temperature of the air at the blade root. Thus, the Mach number calculated is indicative of the actual Mach number but not exactly equal to it. Because the actual maximum Mach number of the cooling air occurs at some undetermined point along the blade where conditions of pressure, temperature, and velocity are critical, exact calculation is extremely difficult; the foregoing approximation is sufficiently accurate for the purposes of this report.

Blade metal. - The blade metal S-497 selected for the application of the analysis to the three turbines is a modern, forged, high-temperature alloy having a strength representative of the better alloys now in use for gas-turbine blades. Strength data for this metal were

taken from reference 7 and extrapolated as shown in figure 4, where the circled points indicate actual data points taken from reference 7. The data give the stresses that the material can withstand for 1000 hours without fracture at various temperatures; therefore, all results shown in the report are for this value of blade life. The method is equally valid for any other blade life; the effectiveness of cooling calculated for a life of 1000 hours, however, is only slightly different from that for any other life, as indicated by the similarity of the two curves in figure 4.

RESULTS AND DISCUSSION

The blade-temperature distribution is given in equation (1) as a function of the distance from the root divided by the length in terms of S and Q and the effective gas temperature t_e. Figure 5 shows plots of this equation for various values of S and Q for $t_e = 1700^{\circ}$ F and plotted for a ratio of blade length to tip radius $L/r_t = 0.38$. Temperature distributions are also shown for a single value of S for three other values of t_e. Corresponding curves in figure 6 show temperature distribution of the cooling air. The temperature at the root of the blade is dependent only on S for a given value of t_e; whereas both S and Q determine the temperature rise along the blade. An analysis of equation (1) shows that for a given value of Q the temperature gradient at the blade root is a maximum for $S = 1.67$. As S decreases below this value, the temperature gradient decreases because of the increase in the mass flow of cooling air absorbing heat; as S increases above this value, the temperature gradient decreases because of the decrease in internal heat-transfer coefficient and the consequent decrease in the amount of heat absorbed. Over the usual range of values of S, Q is the predominant factor in determining the slope of the temperature-distribution curve. The allowable metal temperature at any point along the blade for a variety of tip speeds is also plotted in figure 5. It is evident that the critical point is at the root of the blade except for low limiting speeds and high gas temperatures; therefore, because the normal range of Q is covered by figure 5, Q ordinarily has no effect on the limiting tip speed. Effective hot-gas temperature t_e is plotted in figure 7 against the reciprocal of the cooling variable $1/S$ with limiting tip Mach number index M_{max} as a parameter. This is a general curve depending only on L/r_t and shows that the lower the limiting tip Mach number index, the greater the increase in effective temperature possible for a constant amount of cooling. This curve also shows that increasing the cooling-air flow brings somewhat diminishing returns in effective temperature possible but indicates no definite limit to the practicable amount of cooling.

The amount of cooling that can be obtained with a given turbine, and thus the possible increase in effective gas temperature, may be limited by any one of three factors. These factors are: dilution, that is, the ratio of the cooling-air mass flow to the mass flow of hot gases; the Mach number of the cooling-air flow in the blade; and the pressure drop through the blade. Dilution is a measure of the cooling power required; the cooling air must be compressed if it exhausts into the hot-gas stream. Furthermore, in multistage turbines the exhausting of the cooling air into the hot-gas stream reduces the temperature of the gas entering succeeding stages. For these reasons, there is an optimum amount of cooling, beyond which a gain in gas temperature due to increasing the cooling-air flow becomes uneconomical. Generally, it is expected that this point is located where the dilution is less than about 10 percent but, for any specific turbine, performance calculations would have to be made to determine the optimum dilution. When the cooling-air Mach number reaches unity somewhere in the blade or at the exit, the cooling-air mass flow cannot be further increased because of choking. The pressure drop of the cooling air through the blades is of the same order of magnitude as the increase in pressure due to centrifugal forces on the air, roughly 1 to 3 pounds per square inch; therefore, the pressure drop is of little importance as a limiting factor.

A plot of the increase in effective hot-gas temperature against dilution, with lines of constant cooling-air Mach number superimposed, for the three representative turbines is presented in figure 8. These curves, which show the relation between dilution and Mach number, indicate that dilution is generally the limiting factor. With a dilution of 10 percent as a limit, an increase in effective gas temperature from 150° to 650° F could be expected, depending on the turbine design and the limiting tip Mach number index.

The relation of dilution and Mach number to the variable S, which determines the cooling effectiveness, is shown in equations (4) and (5), respectively. The use of S as a variable (fig. 7) results in a curve that is independent of turbine dimensions and flow conditions with the exception of the slight effect of the ratio of blade length to tip radius L/r_t. The curves of figure 8, in which the limiting factors of dilution and Mach number take the place of S, apply only to particular turbines and show that turbine dimensions and flow conditions have considerable effect on the amount of cooling that can be obtained within given limitations. The effect of specific turbine dimensions and gas-flow conditions can be determined by breaking down these factors into their component parts, as in equations (4) and (5).

The dimensions of the cooling-air passage, described by α, the ratio of internal to external cross-sectional area, and β, the ratio of internal to external perimeter, may be varied to increase the cooling effectiveness. A decrease in α results in a large reduction in cooling-air flow, and thus in dilution, for a given cooling effectiveness but causes only a very small decrease in cooling-air Mach number. An increase in β reduces both the dilution and the cooling-air Mach number. In order to obtain effective cooling therefore, it is desirable that α be small and β be large but their values are interrelated and are limited by mechanical considerations. The value of α may be decreased by decreasing the cooling-air passage in cross-sectional area or by adding filler pieces but, if it is made too small, the cooling-air Mach number may reach an excessive value. In addition, for small values of α the temperature drop through the blade wall cannot be neglected as was done in the analysis; however, with a small value of α there may be some heat flow from the blade to the wheel rim, which is helpful. The value of β may be increased by changing the shape of the passage to increase the inside perimeter by the addition of small fins on the inner surface, by artificial surface roughening, or by a combination of these methods. Any of these methods is mechanically difficult for small blades and, inasmuch as the assumption that heat transfer is directly proportional to the surface area may no longer hold true, an effective area instead of the actual area may have to be used in the calculations.

The ratio of the blade cross-sectional area to the hot-gas-flow area per blade strongly affects the relation between dilution and S and a low value of this ratio is desirable as a means of decreasing the dilution because this ratio has little effect on M_a. A low value of this ratio is obtained with long thin blades of low solidity. Either h_o or G_g, on which h_o is assumed to depend, has a slight effect on dilution and an important effect on M_a. A low value of h_o is desirable for a low value of M_a but a high value is desirable for low dilution. The factors considered in this paragraph are fixed for a given turbine design, however, and serve only to determine how effectively a specific turbine may be air-cooled.

Curves similar to those of figure 8 may be drawn for any turbine by a combination of figure 7 and equations (4) and (5). In general, such curves must not be considered as giving exact quantitative results because figure 7 depends upon the stress-rupture properties for a particular blade material and life and a value of 0.38 for L/r_t. Such curves are, however, sufficiently accurate for finding the effect of variations in turbine dimensions and flow conditions. The optimum method of applying air cooling to any particular turbine may be found

by considering a variety of coolant passages, calculating α and β for each, and then drawing curves similar to those of figure 8 for each. The optimum coolant mass velocity G_a is found by substituting the value of S corresponding to the chosen limiting Mach number or dilution into equation (2). The limiting dilution, however, can only be selected after making a complete cycle analysis of the compressor-turbine unit for various dilutions.

Generally, turbines that can be effectively air-cooled cannot be effectively cooled by rim cooling and vice versa because efficient cooling by these two methods requires two widely different types of blade. The optimum conditions for rim cooling are a relatively short blade with a large cross-sectional area and a small mass velocity of hot gas. On the other hand, if dilution is the limit to the amount of cooling, the best conditions for air cooling are a relatively long blade with a small cross-sectional area and a high value of the mass velocity of the hot gas. Thus, of the three representative turbines used in this report, turbine C shows poor cooling effectiveness as a hollow-bladed turbine but could be effectively rim-cooled if solid blades are used, whereas turbines A and B can be effectively cooled by air cooling of hollow blades.

CONCLUSIONS

An analysis of the air cooling of hollow turbine blades and an application of the analysis to three typical turbines yields the following conclusions:

With a cooling-air flow that is 10 percent of the hot-gas flow, increases in the effective gas temperature from 150° to 650° F are permissible, depending on the dimensions of the turbine and of the cooling-air passages and on the turbine design speed. A turbine specifically designed for air cooling should make possible increases in gas temperature greater than 650° F. Turbines with long thin blades and large hot-gas flows, which cannot be effectively cooled by rim cooling, can be effectively cooled by air cooling.

The effectiveness of cooling is also dependent on the allowable percentage of dilution of the hot gas by the cooling air, because this factor generally limits the amount of cooling. In order to choose the limiting value of the dilution in practice, a complete cycle analysis of a compressor-turbine unit for various cooling flows should be made.

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APPENDIX A

DEFINITION OF SYMBOLS

The following symbols are used in the analysis:

A	blade cross-sectional area, sq ft
A_b	minimum hot-gas flow area per blade, sq ft
a_a	critical velocity of cooling air, ft/sec
a_g	sonic velocity of hot gases at temperature t_e, ft/sec
B	number of blades
C	constant ($C = 0.0323$ Btu/(°F)(hr$^{0.315}$)(ft$^{0.315}$)(lb$^{0.685}$))
c_p	specific heat of cooling air at constant pressure, Btu/(lb)(°F)
D_i	hydraulic diameter of cooling-air passage $4A_i/p_i$, ft
G	mass velocity relative to blades, lb/(hr)(sq ft)
g	acceleration of gravity, 32.2 ft/sec^2
H	heat flow, Btu/hr
h	heat-transfer coefficient, Btu/(hr)(sq ft)(°F)
k_a	thermal conductivity of cooling air, Btu/(hr)(sq ft)(°F/ft)
L	blade length, ft
M_a	Mach number of cooling air based on critical velocity, V_a/a_a
M_{max}	limiting tip Mach number index, V_{max}/a_g
m	mass flow per blade, lb/hr
P	total pressure of cooling air, lb/sq ft
p	blade cross-sectional perimeter, ft

Q	turbine parameter defined in equation (12)
R	gas constant, 53.5 (ft-lb)/(lb)(°R)
r	radius of turbine at any point on blade, ft
r_t	tip radius of turbine, ft
S	cooling variable, $h_o p_o / h_i p_i$
s	blade stress due to centrifugal force, lb/sq in
T_a	total temperature of cooling air, °R
t_a	effective temperature of cooling air, °F
t_{ar}	effective temperature of cooling air at blade root, °F
t_e	effective hot-gas temperature for use in heat-transfer equations, °F
t_m	blade-metal temperature, °F
U	over-all heat-transfer coefficient, Btu/(hr)(sq ft)(°F)
V	blade tip speed, ft/sec
V_a	velocity of cooling air within blade, ft/sec
V_{max}	limiting blade tip speed, ft/sec
w	maximum blade thickness, ft
x	distance from blade root to any point on blade, ft
Y	turbine parameter defined in equation (11), $(\text{lb}/(\text{sq ft})(\text{hr}))^{0.8}$
α	cross-sectional area ratio, A_i/A_o
β	cross-sectional perimeter ratio, p_i/p_o
γ	ratio of specific heats
ϵ	ratio of cooling-air static density to total density

$$\epsilon = \left(1 - \frac{\gamma - 1}{\gamma + 1} M_a^2\right)^{\frac{1}{\gamma-1}}$$

μ_a absolute viscosity of cooling air, lb/hr-ft

ρ density, lb/cu ft

Subscripts:

a cooling air

g hot gas

i inside blade surface

m blade metal

o outside blade surface

APPENDIX B

EQUATION FOR TEMPERATURE DISTRIBUTION

A hollow blade of uniform cross section with thin walls is considered (fig. 1). The temperature gradient through the wall is negligible and circumferential or radial heat flow is ignored.

The fundamental heat-transfer equation is

$$dH = U(t_e - t_a)\ p_o dx = h_o(t_e - t_m)\ p_o dx = h_i(t_m - t_a)\ p_i dx = m_a c_p dt_a \qquad (6)$$

When

$$\frac{\frac{1}{p_o}}{\frac{1}{h_o p_o} + \frac{1}{h_i p_i}} = \frac{h_o}{\frac{h_o p_o}{h_i p_i} + 1}$$

is substituted for U in equation (6),

$$m_a c_p dt_a = \frac{h_o p_o}{\frac{h_o p_o}{h_i p_i} + 1} (t_e - t_a)\ dx$$

If this equation is rearranged and integrated

$$-\frac{m_a c_p}{h_o p_o}\left(\frac{h_o p_o}{h_i p_i} + 1\right) \log_e\left(\frac{t_e - t_a}{t_e - t_{ar}}\right) = x$$

When the foregoing equation is solved for $t_e - t_a$

$$t_e - t_a = (t_e - t_{ar})\ e^{\left(-\frac{h_o p_o x}{m_a c_p} \frac{1}{\frac{h_o p_o}{h_i p_i} + 1}\right)} \qquad (7)$$

In order to express equation (7) in terms of t_m, equation (6) is solved for t_m

$$t_m = \frac{\left(\frac{h_o p_o}{h_i p_i}\right) t_e + t_a}{\frac{h_o p_o}{h_i p_i} + 1}$$

$$t_e - t_m = (t_e - t_a) \frac{1}{\frac{h_o p_o}{h_i p_i} + 1} \tag{8}$$

and equation (7) is substituted in equation (8)

$$t_e - t_m = \frac{1}{\frac{h_o p_o}{h_i p_i} + 1} (t_e - t_{ar}) \, e^{-\left[\frac{h_o p_o x}{m_a c_p}\left(\frac{1}{\frac{h_o p_o}{h_i p_i} + 1}\right)\right]} \tag{9}$$

McAdams (reference 6) gives an equation for the film coefficient for air flowing inside circular and rectangular tubes for Reynolds numbers exceeding 2100. If this equation is assumed to apply to air flowing in passages of irregular shape within turbine blades

$$h_i = 0.020 \left(\frac{D_i G_a}{\mu_a}\right)^{0.8} \frac{k_a}{D_i}$$

For air at a temperature of 300° F

$$\frac{c_p \mu_a}{k_a} = 0.74$$

$$\mu_a = 0.0556 \text{ lb/ hr-ft}$$

$$c_p = 0.25 \text{ Btu/(lb)(}^{\circ}\text{F)}$$

Then, if the variation of h_i due to the slight change in $(\mu_a)^{0.2}$ and c_p with temperature is neglected,

$$h_i = 0.00286 \left(\frac{p_i}{A_i}\right)^{0.2} G_a^{0.8} \tag{10}$$

In order to reduce equation (9) to a more usable form, the variables S, α, and β are introduced

$$S = \frac{h_o p_o}{h_i p_i}$$

$$\alpha = \frac{A_i}{A_o}$$

$$\beta = \frac{p_i}{p_o}$$

If equation (10) is substituted in the expression defining S

$$S = \frac{h_o p_o}{h_i p_i} = \left[350 \; h_o \left(\frac{1}{\beta}\right)^{1.2} \alpha^{0.2} \left(\frac{A_o}{p_o}\right)^{0.2}\right] \left(\frac{1}{G_a}\right)^{0.8} = Y \; G_a^{-0.8}$$

where

$$Y = 350 \; h_o \left(\frac{1}{\beta}\right)^{1.2} \alpha^{0.2} \left(\frac{A_o}{p_o}\right)^{0.2} \tag{11}$$

When m_a is taken as $G_a A_i$ and c_p as 0.25 in the exponent of e in equation (9)

$$\frac{h_o p_o x}{m_a c_p} = 4 \left(\frac{1}{G_a}\right) \left(\frac{1}{\alpha}\right) L \left(\frac{x}{L}\right) \frac{h_o p_o}{A_o} = S^{1.25} \; Q \left(\frac{x}{L}\right)$$

where

$$Q = 0.002644 \; h_o^{-0.25} \; \beta^{1.5} \; p_o^{1.25} \; \alpha^{-1.25} \; A_o^{-1.25} \; L \qquad (12)$$

and equation (9) may be written

$$t_e - t_m = \frac{1}{S + 1} (t_e - t_{ar}) \; e^{-\left[S^{1.25} \; Q \left(\frac{1}{S + 1}\right) \frac{x}{L}\right]} \qquad (1)$$

APPENDIX C

EQUATIONS FOR DILUTION AND COOLING-AIR MACH NUMBER

A curve for determining the heat-transfer coefficient for turbine buckets was supplied by the General Electric Company. Although this curve was found for only one set of impulse blades, it is used in this report for all blades because it approximately agrees with other data for flow over plates and cylinders and no other data on blades are available. The equation of this curve is

$$h_o = C\, G_g^{0.685}\, p_o^{-0.315} \tag{13}$$

Because the value of G_g at the smallest flow area is to be used with the data supplied from the General Electric Company, the mass flow of hot gases per blade is given as

$$m_g = G_g A_b \tag{14}$$

where A_b, the minimum hot-gas-flow area per blade, may be expressed

$$A_b = \left[2\pi\left(r_t - \frac{L}{2}\right) - B\, w\right] L/B$$

When m_a/A_i is substituted for G_a in equation (10)

$$h_i = 0.00286\, p_i^{0.2}\, A_i^{-1}\, m_a^{0.8} \tag{15}$$

An expression for S in terms of dilution is then found by using equations (13), (14), and (15) together with the definition of S

$$S = 350\, C G_g^{-0.115} \left(\frac{m_g}{m_a}\right)^{0.8} \alpha A_o^{0.2}\, \beta^{-1.2}\, p_o^{-0.515} \left(\frac{A_o}{A_b}\right)^{0.8} \tag{16}$$

When equation (16) is solved for m_a/m_g,

$$\frac{m_a}{m_g} = 1512\, S^{-1.25}\, C^{1.25}\, G_g^{-0.144}\left(\frac{A_o}{A_b}\right) \alpha^{1.25}\, A_o^{0.25}\, \beta^{-1.5}\, p_o^{-0.644} \tag{4}$$

The cooling-air Mach number M_a may also be related to the cooling variable S if the cooling-air mass velocity G_a is expressed in terms of M_a:

$$G_a = 3600\ \rho_a V_a = 3600\ \frac{P}{RT_a}\ \epsilon M_a \sqrt{\frac{2\gamma}{\gamma+1}\ gRT_a} = 3600\ P \sqrt{\frac{2\gamma}{(\gamma+1)}\ \frac{g}{RT_a}}\ \epsilon M_a \qquad (17)$$

Equations (11) and (17) are substituted in equation (2) giving an expression for S in terms of M_a

$$S = 0.498\ h_o \alpha^{0.2}\ A_o^{0.2}\ \beta^{-1.2}\ p_o^{-0.2}\ P^{-0.8}$$

$$\left[\frac{2\gamma g}{(\gamma+1)\ RT_a}\right]^{-0.4} (\epsilon M_a)^{-0.8} \qquad (18)$$

Because ϵ is a complicated function of M_a (appendix A), equation (18) must be solved for $M_a\epsilon$ rather than M_a:

$$M_a\epsilon = 0.421\ S^{-1.25}\ C^{1.25}\ G_g^{0.856}\ \alpha^{0.25}\ A_o^{0.25}\ \beta^{-1.5}\ p_o^{-0.644}\ P^{-1}\ T_a^{0.5}$$

$$\left[\frac{2\gamma g}{(\gamma+1)\ R}\right]^{-0.5} \qquad (5)$$

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TABLE I - DIMENSIONS OF THREE TURBINES USED IN CALCULATIONS

Turbine	Cross-sectional area of outside blade surface, A_o (sq ft)	Cross-sectional area of cooling passage, A_i (sq ft)	Cross-sectional area ratio, α	Outside perimeter, p_o (ft)	Inside perimeter, p_i (ft)	Cross-sectional perimeter ratio, β
A	0.00198	0.00157	0.792	0.254	0.2335	0.919
B	.00216	.001702	.788	.377	.2605	.691
C	.000286	.000171	.597	.095	.0859	.904

Turbine	Blade length, L (ft)	Tip radius, r_t (ft)	Number of blades, B	Maximum blade thickness, w (ft)	Turbine parameter, Y $(lb/sq\ ft\text{-}hr)^{0.8}$	Turbine parameter, Q	Minimum hot-gas flow area per blade, A_b (sq ft)
A	0.333	0.875	55	0.0252	30,400	0.1230	0.01854
B	.208	.584	30	.0268	15,500	.0903	.01533
C	.1011	.510	142	.0108	9,100	.2048	.00195

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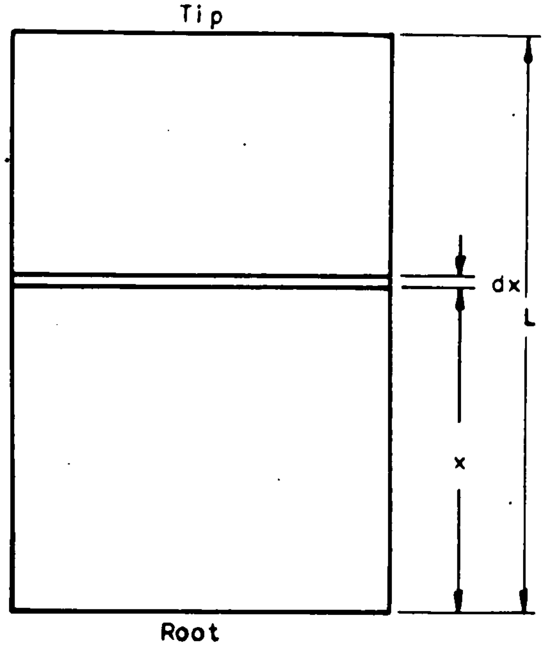

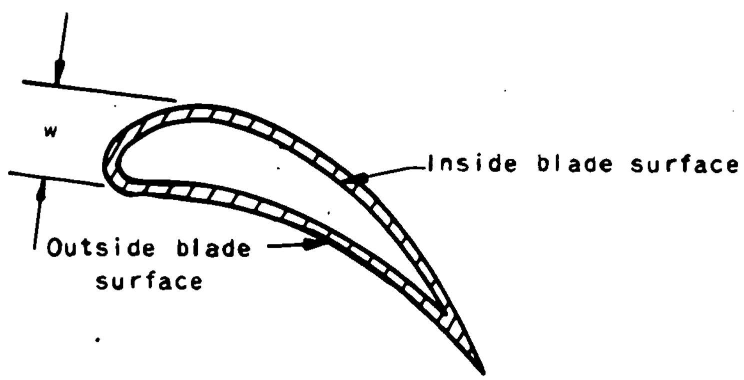

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Figure 1. - Diagram showing cross section and elevation of typical blade.

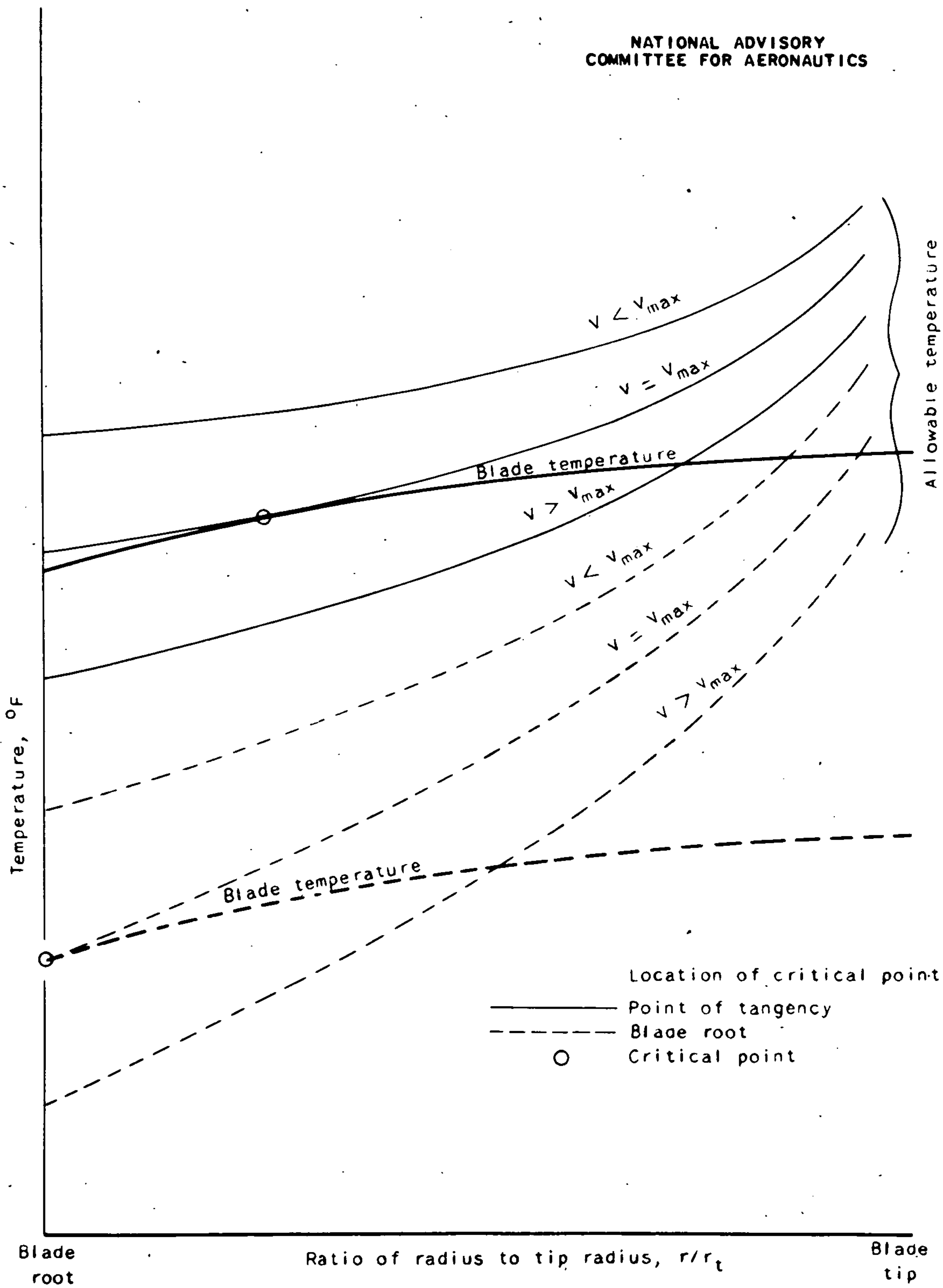

Figure 2. – Schematic diagram showing method of determining limiting blade tip speed V_{max}.

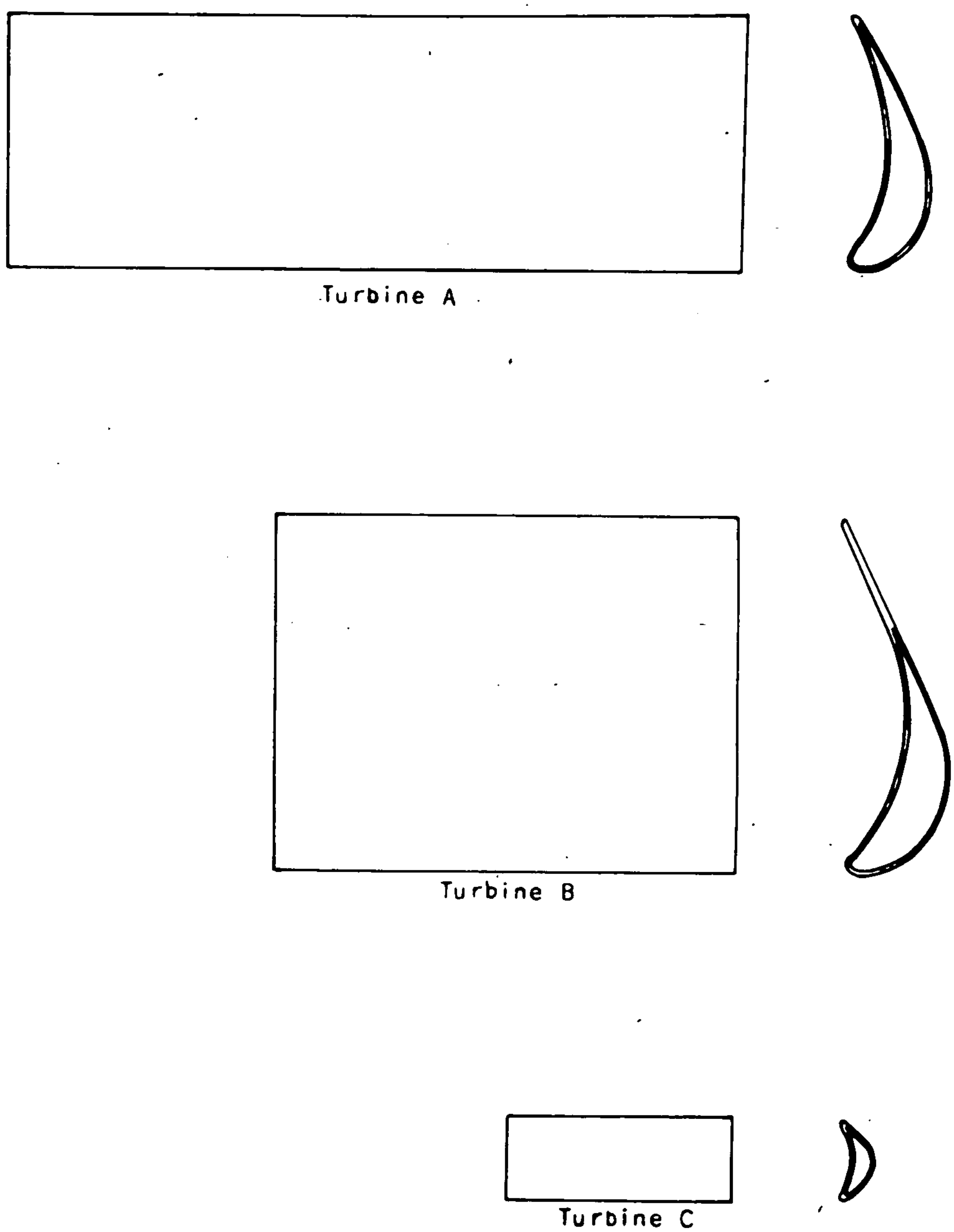

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Figure 3. - Full-scale sketches of blades of three representative turbines used in application of analysis.

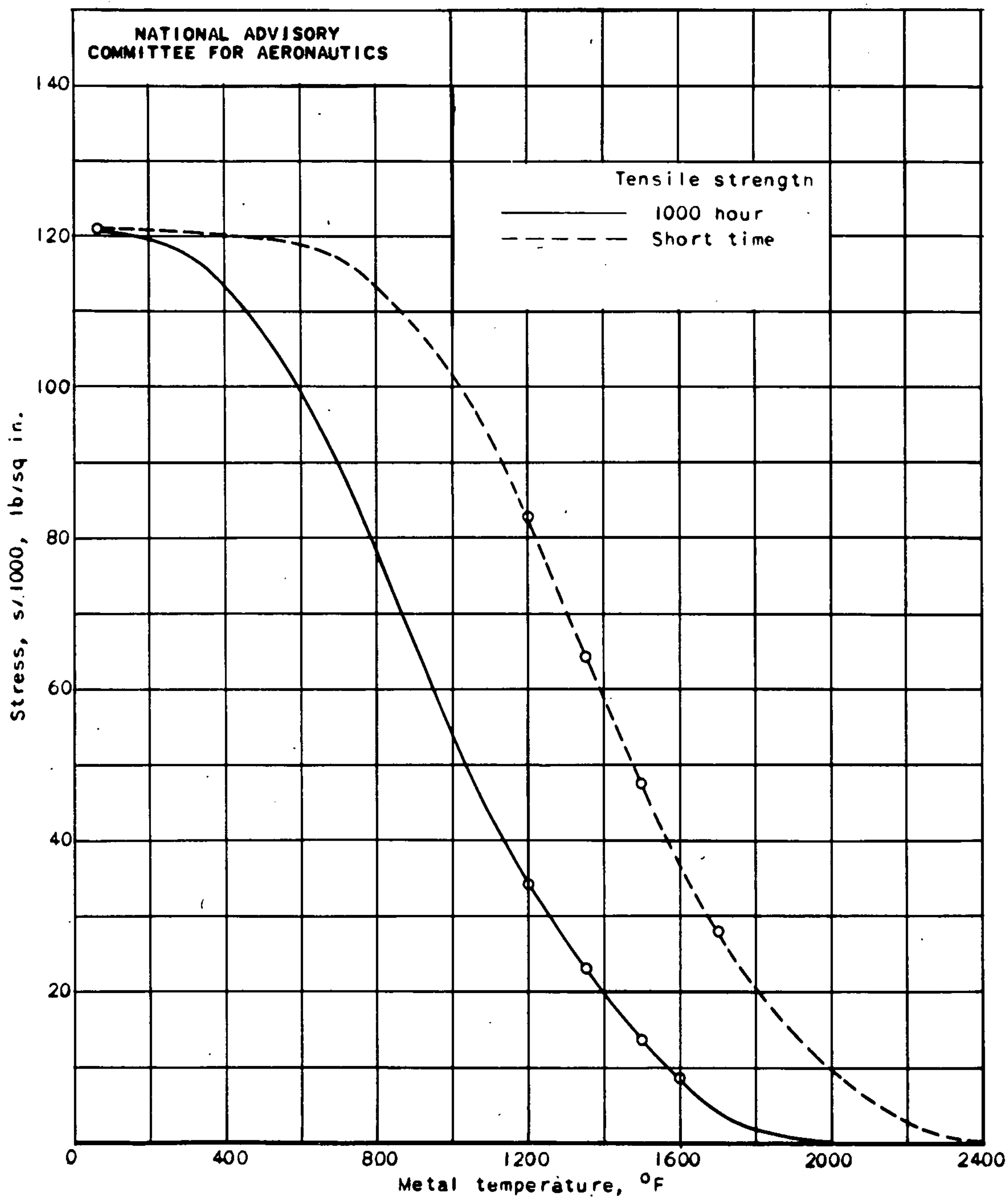

Figure 4. - Strength properties of S-497 metal, showing extrapolation of curve for 1000-hour tensile strength. (Data from reference 7 are indicated by circled points.)

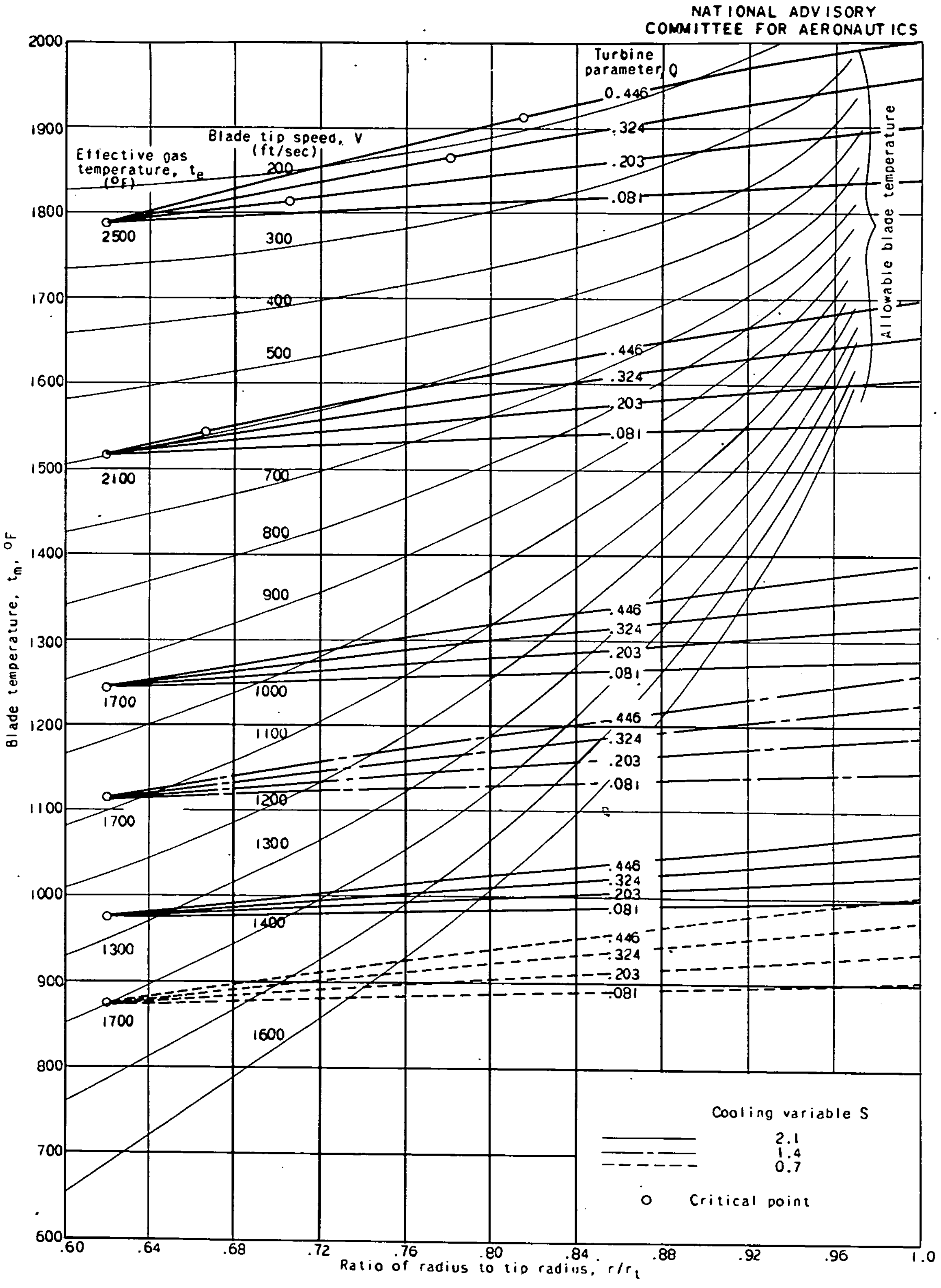

Figure 5. - Blade-temperature distributions (equation (1)) for various values of effective hot-gas temperature t_e, cooling variable S, and turbine parameter Q superimposed on lines of allowable temperature showing location of critical points for 1000-hour blade life and for any turbine with $L/r_t = 0.38$, such as turbine A.

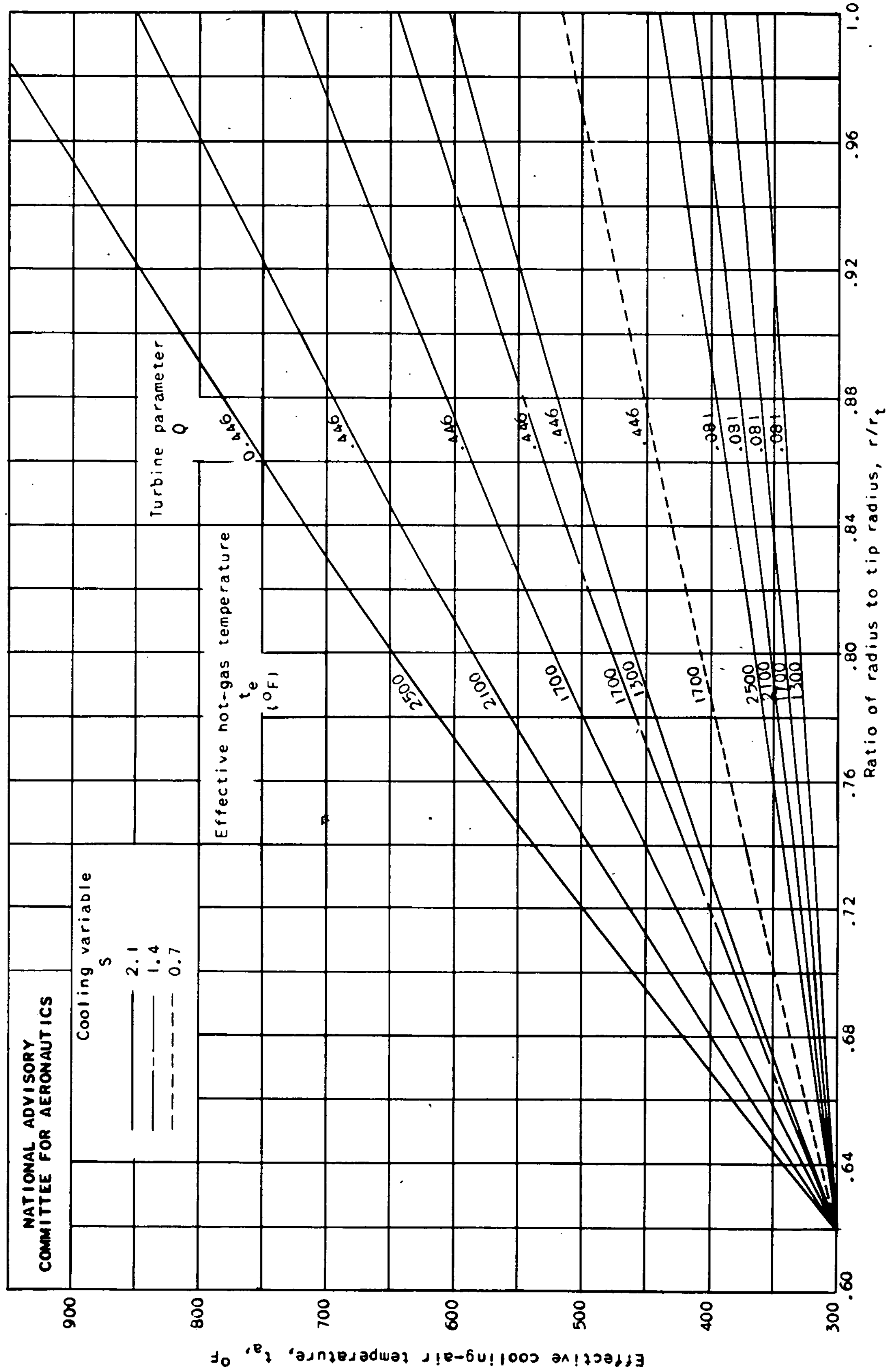

Figure 6. - Cooling-air temperature distribution.

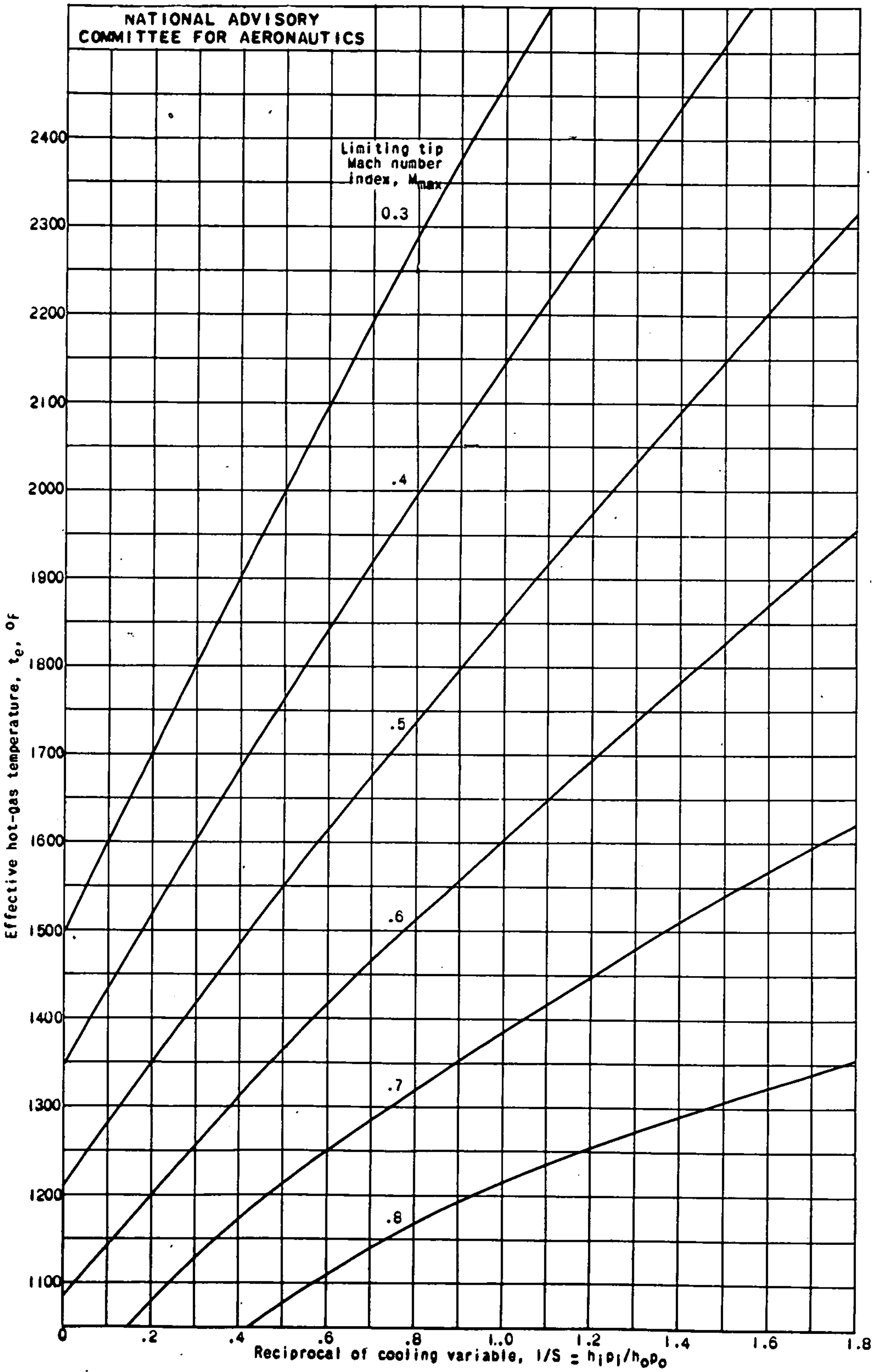

Figure 7. - Variation of effective gas temperature with reciprocal of cooling variable S for constant values of tip Mach number index, $L/r_t = 0.38$.

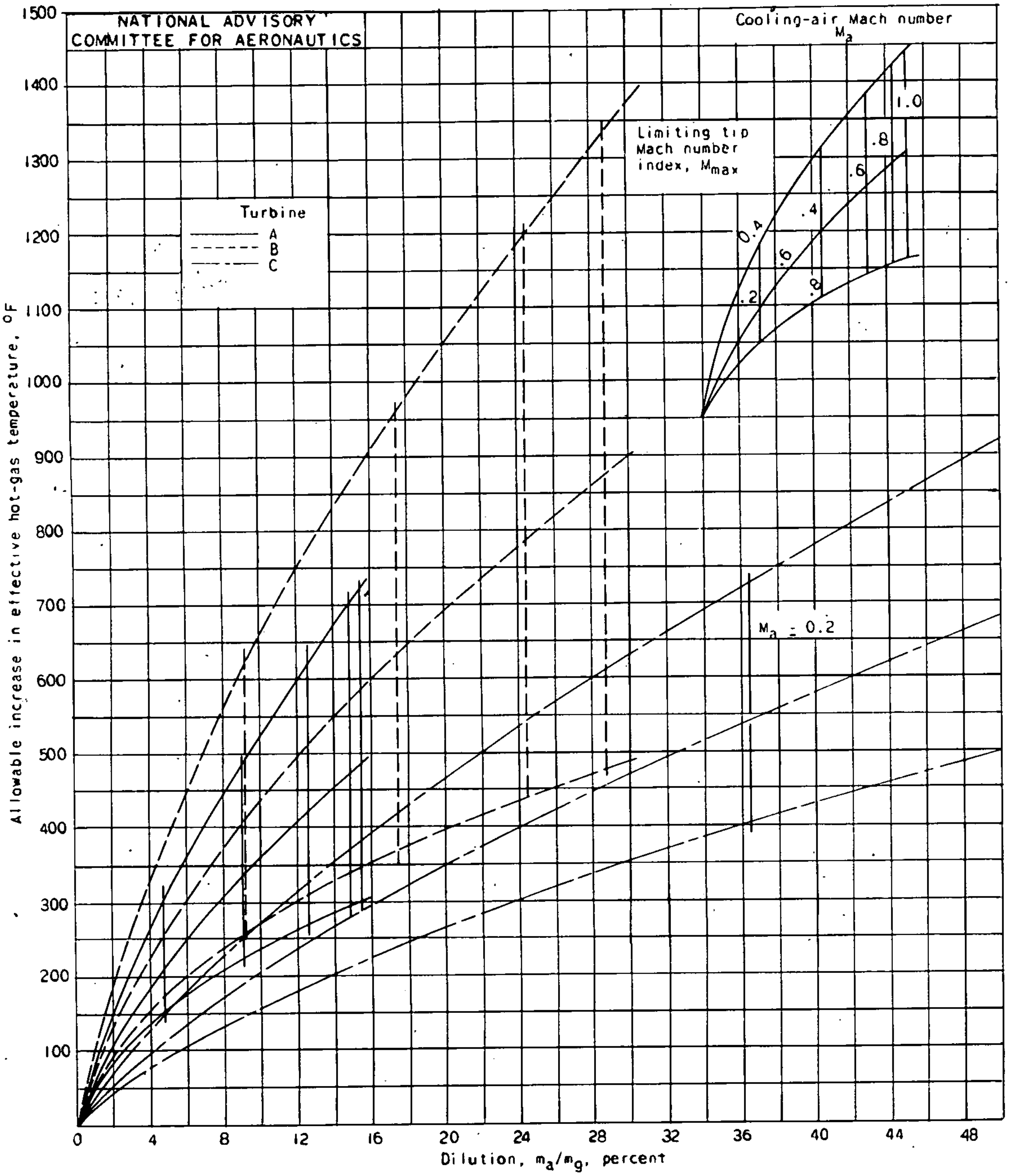

Figure 8. - Comparison of effectiveness of cooling for turbines A, B, and C.